S STUDENTS

National Student Team Contest (first stage)
Solution of task 9. Hydrophobic solar cells

Let's calculate angle, q, which will be formed between surface of the drop and surface of the cell. According to Young equation for contact angle, one can get:

$$\Delta\gamma = \gamma_{LG}\cos\theta \quad , \qquad (1)$$

where g_{LG} is a surface tension coefficient on the bound water-air equal to 72 mN/m.
Then:

$$\theta = \arccos\frac{\Delta\gamma}{\gamma_{LG}} = \frac{\pi}{3} \quad . \qquad (2)$$

So surface area will be equal to p (r sin q)^2, because contact angle precisely equals to the angle of scherical layer cut by the surface of the cell. However the complexity of the question lies on the fact that due to deformability of the drop its new radius, r, does not equal to initial radius, R. Let's find, r, taking into account stable volume of the drop.Volume of the scherical layer equals to:

$$V_{sc} = \frac{\pi}{3}r^3(\cos\theta + 2)(\cos\theta - 1)^2 = \frac{\pi}{3}r^3\frac{5}{2}\frac{1}{4} = \frac{5}{32}\frac{4}{3}\pi r^3 \quad . \qquad (3)$$

Let's put down condition of the equality of drop volumes before and after adhesion on the surface of the cell:

$$V = \frac{4}{3}\pi R^3 = \frac{27}{32}\frac{4}{3}\pi r^3 \qquad (4)$$

Then:

$$r = R\frac{2\sqrt[3]{4}}{3} \qquad (5)$$

Finally contact area equals to:

$$S = \pi(r\sin\theta)^2 = \pi\left(R\sqrt[3]{4}\frac{\sqrt{3}}{3}\right)^2 = \pi(3\sqrt[3]{16}) = 6\pi\sqrt[3]{2} = 23.7\,\mathrm{mm}^2 \quad . \qquad (6)$$